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Perishable Inventory Model with Time Dependent Demand and Partial Backlogging

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Abstract
An Inventory Model For Decaying Items Under Inflation Has Been Developed. Demand Rate Is Taken As Linear Time Dependent. Holding Cost Is Also Taken As Time Dependent. We Have Developed The Two Cases: In The First Case Shortages Are Not Allowed And In The Second Case Partially Backlogged Shortages Are Allowed. Cost Minimization Technique Is Used In This Study.

Keywords: Time dependent demand, partial backlogging, shortages, perishable inventory model.

I. INTRODUCTION

Planning is the art of specifying meaningful information about the future. Long run planning decisions require consideration of many factors: general economic conditions, industry trends, probable competitor actions and so on. For proper management of system resources, there must be an adequate knowledge base of the customer's demand. This demand needs to be managed in a proper manner. The purpose of demand management is to coordinate and control all the sources, so that production and operations systems can be utilized efficiently.As inventory represents a very important part of the company's financial assets, it is very much affected by the market's response to various situations, especially inflation. Inflation is a global phenomenon in present day times. Inflation can be defined as that state of disequilibrium in which an expansion of purchasing power tends to cause or is the effect of an increase in the price level. A period of prolonged, persistent and continuous inflation results in the economic, political, social and moral disruption of society. Almost everyone thinks inflation is evil, but it isn't necessarily so. Inflation affects different people in different ways. It also depends on whether inflation is anticipated or unanticipated. If the inflation rate corresponds to what the majority of people are expecting (anticipated inflation), then we can compensate, and the cost isn't high.

Nowadays inflation has become a permanent feature in the inventory system. Inflation enters in the picture of inventory only because it may have an impact on the present value of the future inventory cost. Thus the inflation plays a vital role in the inventory system and production management though the decision makers may face difficulties in arriving at answers related to decision making. At present, it is impossible to ignore the effects of inflation and it is necessary to consider the effects of inflation on the inventory system.

Buzacott (1975) developed the first EOQ model taking inflationary effects into account. In this model, a uniform inflation was assumed for all the associated costs and an expression for the EOQ was derived by minimizing the average annual cost. **Bierman** and **Thomas (1977)** suggested the inventory decision policy under inflationary conditions.Economic analysis of dynamic inventory models with non-stationary costs and demand was presented by **Hariga (1994)**. The effect of inflation was also considered in this analysis. An economic order quantity inventory model for deteriorating items with inflation was developed by **Bose et al. (1995)**. Effects of inflation and time-value of money on an inventory model was discussed by **Hariga (1995)** with linearly increasing demand rate and shortages. **Hariga** and **Ben-Daya (1996)** then discussed the inventory replenishment problem over a fixed planning horizon for items with linearly time-varying demand under inflationary conditions. A generalized dynamic programming model for inventory items with Weibull distributed deterioration and time dependent demand was proposed by **Chen (1998)**. The effects of inflation and time-value of money on an economic order quantity model have been discussed by **Moon** and **Lee (2000)**.

Chang (2004) proposed an inventory model with the effect of inflation, deteriorating items and delay in payments. Models for ameliorating/deteriorating items with time-varying demand pattern over a finite planning horizon were proposed by **Moon et al. (2005)**. The effects of inflation and time value of money were also taken into account. **Jolai et al. (2006)** presented an optimization framework to derive optimal production over a fixed planning horizon for items with a stock-dependent demand rate under inflationary conditions. **Jaggi et al. (2007)** presented the optimal inventory replenishment policy for deteriorating items under inflationary conditions using a discounted cash flow (DCF) approach over a finite time horizon. Shortages in inventory were allowed and completely backlogged and demand rate was assumed to be a function of inflation. **Chern, M. S. et al. (2008)** developed an inventory lot-size model for deteriorating items with partial backlogging and time value of money. **Roy, A., Pal, S. and Maiti, M.K. (2009)** considered a production inventory model with inflation and time value of money and demand of the item is displayed stock dependent and lifetime of the product is random in nature and follows exponential distribution with a known mean. This paper presents an order level inventory system with non-instantaneous deteriorating items with time dependent demand, where suppliers offer an all unit quantity discount. Two cases have been discussed in this study: in case first shortages are not allowed and in case second shortages are allowed with partially backlogged. The whole study has been taken with inflationary environment. Special cases are discussed in this study. The purpose of this study is to determine an optimal ordering policy for minimizing the expected total relevant inventory cost.

II. ASSUMPTIONS AND NOTATIONS

Assumptions

1. The product life time (time to deterioration) t has a p.d.f. $f(t) = \theta e^{-\theta(t-t_d)}$ for $t > t_d$. So that the deterioration rate is $r(t) = \dfrac{f(t)}{1-F(t)} = \theta$, for $t > t_d$.
2. The demand rate $D(t) = \alpha + \beta t$, where α and β are positive constants.
3. The effect of inflation has been taken.
4. The replenishment rate is finite.
5. Lead time is zero.
6. Shortages are allowed and partial backlogged.
7. There is no replacement or repair of deteriorated units during the period under consideration.

Notations

A Ordering cost per order

q_i The i[th] price breaking point i=0,1,2…..m, where $0 < q_0 < q_1 ... < q_m$

C_i The purchasing cost per unit dependent on order size, i=0,1,2….m,

where $C_0 > C_1 > ... > C_m > 0$.

$(C_1 + ht)$ Linear holding cost

C_S Shortages cost per unit backordered per unit time

C_{LS} The cost of lost sales per unit

$e^{-\delta t}$ Backlogging rate

θ Parameter of the deterioration rate function

t_d The length of time in which the product has no deterioration

t_1 The length of time with positive stock

T The replenishment cycle time, where $T \ge t_1$

Q Order quantity per cycle

ae^{bt} Exponential demand rate

θt Time dependent deterioration rate

Kae^{bt} Demand dependent production

$C_1 t^{\gamma}$ Holding cost per unit per unit time

C_S Shortages cost per unit per unit time

C_{LS} Lost sale cost per unit per unit time

C' Set up cost

III. MATHEMATICAL FORMULATION

This paper is developed the replenishment problem of a single non-instantaneous deteriorating item with partial backlogging and variable holding cost. $I_{\max}$ units of the item arrive at the inventory system at the beginning of each cycle. At, $[0, t_d]$ the inventory level decreases only owing to stock and time dependent demand rate. The inventory level drops to zero due to demand and deterioration of items at $[t_d, t_1]$. The shortage occurs and keeps to the end of the current order cycle. The entire process is repeated.
We consider two cases: first, when shortages are completely backlogged and second, when shortages are partially backlogged.

CASE I: When shortages are not allowed

The inventory level decreases only due to demand during the interval $[t_d, t_1]$. Hence the differential equation representing the inventory level is given by:

$$\frac{dI_1(t)}{dt} = -(\alpha + \beta t) \qquad 0 \le t \le t_d \qquad \ldots(1)$$

With the boundary condition $I_1(0) = I_{\max}$. Solution of equation (1) is:

$$I_1(t) = I_{\max} - (\alpha t + \frac{\beta t^2}{2}) \qquad 0 \le t \le t_d \qquad \ldots(2)$$

$$\frac{dI_2(t)}{dt} + \theta I_2(t) = -(\alpha + \beta t) \qquad t_d \le t \le t_1 \qquad \ldots(3)$$

With the boundary condition $I_2(t_1) = 0$. Solution of equation (3) is:

$$I_2(t) = [\frac{\alpha}{\theta}(e^{\theta(t_1 - t)} - 1) + \frac{\beta}{\theta}(t_1 e^{\theta(t_1 - t)} - t) - \frac{\beta}{\theta^2}(e^{\theta(t_1 - t)} - 1)] \qquad t_d \le t \le t_1 \qquad \ldots(4)$$

Equating the equation (2) and (4) at $t = t_d$

$$I_{\max} - (\alpha t_d + \frac{\beta t_d^{\,2}}{2}) = [\frac{\alpha}{\theta}(e^{\theta(t_1 - t_d)} - t_d) + \frac{\beta}{\theta}(t_1 e^{\theta(t_1 - t_d)} - t_d) - \frac{\beta}{\theta^2}(e^{\theta(t_1 - t_d)} - 1)]$$

This implies that the maximum inventory level for each cycle is:

$$I_{\max} = [\frac{\alpha}{\theta}(e^{\theta(t_1 - t_d)} - t_d) + \frac{\beta}{\theta}(t_1 e^{\theta(t_1 - t_d)} - t_d) - \frac{\beta}{\theta^2}(e^{\theta(t_1 - t_d)} - 1) - (\alpha t_d + \frac{\beta t_d^{\,2}}{2})] \qquad \ldots(5)$$

Substitute the value of $I_{\max}$ from equation (5) in equation (2), we get

$$I_1(t) = [\frac{\alpha}{\theta}(e^{\theta(t_1 - t_d)} - t_d) + \frac{\beta}{\theta}(t_1 e^{\theta(t_1 - t_d)} - t_d) - \frac{\beta}{\theta^2}(e^{\theta(t_1 - t_d)} - 1) - (\alpha t_d + \frac{\beta t_d^{\,2}}{2})] - (\alpha t + \frac{\beta t^2}{2}), \quad 0 \le t \le t_d \quad \ldots(6)$$

From equation (5), we can obtain the order quantity Q as:

$$Q = I_{\max}$$

$$= [\frac{\alpha}{\theta}(e^{\theta(t_1 - t_d)} - t_d) + \frac{\beta}{\theta}(t_1 e^{\theta(t_1 - t_d)} - t_d) - \frac{\beta}{\theta^2}(e^{\theta(t_1 - t_d)} - 1) - (\alpha t_d + \frac{\beta t_d^{\,2}}{2})] \qquad \ldots(7)$$

The total relevant cost per unit time consists of

Ordering cost $O_C = \frac{A}{T}$ …(8)

Inventory holding cost $H_C = \frac{1}{T}[\int_0^{t_d} C_1 I(t) e^{-rt} dt + \int_{t_d}^{t_1} C_1 I(t) e^{-rt} dt]$

$$= \frac{1}{T}[C_1\{\frac{\alpha}{\theta}(e^{\theta(t_1 - t_d)} - t_d)t_d + \frac{\beta}{\theta}(t_1 e^{\theta(t_1 - t_d)} - t_d)t_d - \frac{\beta}{\theta^2}(e^{\theta(t_1 - t_d)} - 1)t_d$$

$$-(\frac{3\alpha t_d^{\,2}}{2} + \frac{2\beta t_d^{\,3}}{3}) - \frac{r\alpha t_d^{\,2}}{2\theta}(e^{\theta(t_1 - t_d)} - t_d) - \frac{r\beta t_d^{\,2}}{2\theta}(t_1 e^{\theta(t_1 - t_d)} - t_d)$$

$$-\frac{r\beta t_d^{\,2}}{2\theta^2}(e^{\theta(t_1-t_d)}-1)-\frac{rt_d^{\,2}}{2}(\alpha t_d+\frac{\beta t_d^{\,2}}{2})+r(\frac{\alpha t_d^{\,3}}{3}+\frac{\beta t_d^{\,4}}{8})\}$$
$$+\{\frac{\alpha}{\theta}(-\frac{1}{\theta}+\frac{e^{\theta(t_1-t_d)}}{\theta}-(t_1-t_d))+\frac{\beta}{\theta}(-\frac{t_1}{\theta}+\frac{t_1e^{\theta(t_1-t_d)}}{\theta}-\frac{t_1^{\,2}}{2}+\frac{t_d^{\,2}}{2})$$
$$-\frac{\beta}{\theta^2}(-\frac{1}{\theta}+\frac{e^{\theta(t_1-t_d)}}{\theta}-(t_1-t_d))-\frac{r\alpha}{\theta}(-\frac{t_1}{\theta}+\frac{t_de^{\theta(t_1-t_d)}}{\theta}-\frac{1}{\theta^2}+\frac{e^{\theta(t_1-t_d)}}{\theta^2}$$
$$-\frac{t_1^{\,2}}{2}+\frac{t_d^{\,2}}{2})-\frac{r\beta}{\theta}(-\frac{t_1^{\,2}}{\theta}+\frac{t_1t_de^{\theta(t_1-t_d)}}{\theta}-\frac{t_1}{\theta^2}+\frac{t_1e^{\theta(t_1-t_d)}}{\theta^2}-\frac{t_1^{\,3}}{3}+\frac{t_d^{\,3}}{3})$$
$$+\frac{r\beta}{\theta^2}(-\frac{t_1}{\theta}+\frac{t_de^{\theta(t_1-t_d)}}{\theta}-\frac{1}{\theta^2}+\frac{e^{\theta(t_1-t_d)}}{\theta^2}-\frac{t_1^{\,2}}{2}+\frac{t_d^{\,2}}{2})\}] \qquad \ldots(9)$$

Inventory deterioration cost $D_C=\frac{C_d}{T}[\int_{t_d}^{t_1}\theta I(t)e^{-rt}dt]$

$$=\frac{C_d\theta}{T}[\{\frac{\alpha}{\theta}(-\frac{1}{\theta}+\frac{e^{\theta(t_1-t_d)}}{\theta}-(t_1-t_d))+\frac{\beta}{\theta}(-\frac{t_1}{\theta}+\frac{t_1e^{\theta(t_1-t_d)}}{\theta}-\frac{t_1^{\,2}}{2}+\frac{t_d^{\,2}}{2})$$
$$-\frac{\beta}{\theta^2}(-\frac{1}{\theta}+\frac{e^{\theta(t_1-t_d)}}{\theta}-(t_1-t_d))-\frac{r\alpha}{\theta}(-\frac{t_1}{\theta}+\frac{t_de^{\theta(t_1-t_d)}}{\theta}-\frac{1}{\theta^2}+\frac{e^{\theta(t_1-t_d)}}{\theta^2}$$
$$-\frac{t_1^{\,2}}{2}+\frac{t_d^{\,2}}{2})-\frac{r\beta}{\theta}(-\frac{t_1^{\,2}}{\theta}+\frac{t_1t_de^{\theta(t_1-t_d)}}{\theta}-\frac{t_1}{\theta^2}+\frac{t_1e^{\theta(t_1-t_d)}}{\theta^2}-\frac{t_1^{\,3}}{3}+\frac{t_d^{\,3}}{3})$$
$$+\frac{r\beta}{\theta^2}(-\frac{t_1}{\theta}+\frac{t_de^{\theta(t_1-t_d)}}{\theta}-\frac{1}{\theta^2}+\frac{e^{\theta(t_1-t_d)}}{\theta^2}-\frac{t_1^{\,2}}{2}+\frac{t_d^{\,2}}{2})\}] \qquad \ldots(10)$$

Purchase cost $P_C=C_iQ$

$$=C_i[\frac{\alpha}{\theta}(e^{\theta(t_1-t_d)}-t_d)+\frac{\beta}{\theta}(t_1e^{\theta(t_1-t_d)}-t_d)-\frac{\beta}{\theta^2}(e^{\theta(t_1-t_d)}-1)-(\alpha t_d+\frac{\beta t_d^{\,2}}{2})] \qquad \ldots(11)$$

The total relevant inventory cost per unit time can be formulated as:

$TVC(t_1,T)=O_C+H_C+D_C+P_C$

$$=\frac{A}{T}+\frac{1}{T}[C_1\{\frac{\alpha}{\theta}(e^{\theta(t_1-t_d)}-t_d)t_d+\frac{\beta}{\theta}(t_1e^{\theta(t_1-t_d)}-t_d)t_d-\frac{\beta}{\theta^2}(e^{\theta(t_1-t_d)}-1)t_d$$
$$-(\frac{3\alpha t_d^{\,2}}{2}+\frac{2\beta t_d^{\,3}}{3})-\frac{r\alpha t_d^{\,2}}{2\theta}(e^{\theta(t_1-t_d)}-t_d)-\frac{r\beta t_d^{\,2}}{2\theta}(t_1e^{\theta(t_1-t_d)}-t_d)$$
$$-\frac{r\beta t_d^{\,2}}{2\theta^2}(e^{\theta(t_1-t_d)}-1)-\frac{rt_d^{\,2}}{2}(\alpha t_d+\frac{\beta t_d^{\,2}}{2})+r(\frac{\alpha t_d^{\,3}}{3}+\frac{\beta t_d^{\,4}}{8})\}$$
$$+\{\frac{\alpha}{\theta}(-\frac{1}{\theta}+\frac{e^{\theta(t_1-t_d)}}{\theta}-(t_1-t_d))+\frac{\beta}{\theta}(-\frac{t_1}{\theta}+\frac{t_1e^{\theta(t_1-t_d)}}{\theta}-\frac{t_1^{\,2}}{2}+\frac{t_d^{\,2}}{2})$$
$$-\frac{\beta}{\theta^2}(-\frac{1}{\theta}+\frac{e^{\theta(t_1-t_d)}}{\theta}-(t_1-t_d))-\frac{r\alpha}{\theta}(-\frac{t_1}{\theta}+\frac{t_de^{\theta(t_1-t_d)}}{\theta}-\frac{1}{\theta^2}+\frac{e^{\theta(t_1-t_d)}}{\theta^2}$$
$$-\frac{t_1^{\,2}}{2}+\frac{t_d^{\,2}}{2})-\frac{r\beta}{\theta}(-\frac{t_1^{\,2}}{\theta}+\frac{t_1t_de^{\theta(t_1-t_d)}}{\theta}-\frac{t_1}{\theta^2}+\frac{t_1e^{\theta(t_1-t_d)}}{\theta^2}-\frac{t_1^{\,3}}{3}+\frac{t_d^{\,3}}{3})$$
$$+\frac{r\beta}{\theta^2}(-\frac{t_1}{\theta}+\frac{t_de^{\theta(t_1-t_d)}}{\theta}-\frac{1}{\theta^2}+\frac{e^{\theta(t_1-t_d)}}{\theta^2}-\frac{t_1^{\,2}}{2}+\frac{t_d^{\,2}}{2})\}]$$
$$+\frac{C_d\theta}{T}[\{\frac{\alpha}{\theta}(-\frac{1}{\theta}+\frac{e^{\theta(t_1-t_d)}}{\theta}-(t_1-t_d))+\frac{\beta}{\theta}(-\frac{t_1}{\theta}+\frac{t_1e^{\theta(t_1-t_d)}}{\theta}-\frac{t_1^{\,2}}{2}+\frac{t_d^{\,2}}{2})$$

$$-\frac{\beta}{\theta^2}(-\frac{1}{\theta}+\frac{e^{\theta(t_1-t_d)}}{\theta}-(t_1-t_d))-\frac{r\alpha}{\theta}(-\frac{t_1}{\theta}+\frac{t_d e^{\theta(t_1-t_d)}}{\theta}-\frac{1}{\theta^2}+\frac{e^{\theta(t_1-t_d)}}{\theta^2}$$
$$-\frac{t_1^{\ 2}}{2}+\frac{t_d^{\ 2}}{2})-\frac{r\beta}{\theta}(-\frac{t_1^{\ 2}}{\theta}+\frac{t_1 t_d e^{\theta(t_1-t_d)}}{\theta}-\frac{t_1}{\theta^2}+\frac{t_1 e^{\theta(t_1-t_d)}}{\theta^2}-\frac{t_1^{\ 3}}{3}+\frac{t_d^{\ 3}}{3})$$
$$+\frac{r\beta}{\theta^2}(-\frac{t_1}{\theta}+\frac{t_d e^{\theta(t_1-t_d)}}{\theta}-\frac{1}{\theta^2}+\frac{e^{\theta(t_1-t_d)}}{\theta^2}-\frac{t_1^{\ 2}}{2}+\frac{t_d^{\ 2}}{2})\}]$$
$$+C_i[\frac{\alpha}{\theta}(e^{\theta(t_1-t_d)}-t_d)+\frac{\beta}{\theta}(t_1 e^{\theta(t_1-t_d)}-t_d)-\frac{\beta}{\theta^2}(e^{\theta(t_1-t_d)}-1)-(\alpha t_d+\frac{\beta t_d^{\ 2}}{2})] \quad \ldots(12)$$

To minimize the total average cost per unit of time, the optimal value of t_1 and T can be obtained by the following equations

$$\frac{\partial TVC(t_1,T)}{\partial t_1}=0 \quad \text{and} \quad \frac{\partial TVC(t_1,T)}{\partial T}=0$$
$$\frac{\partial^2 TVC(t_1,T)}{\partial^2 t_1}>0 \quad \text{and} \quad \frac{\partial^2 TVC(t_1,T)}{\partial^2 T}>0$$

SPECIAL CASES

CASE A: When $\beta=0$, demand rate is constant. Then the total relevant inventory cost per unit time can be formulated as:

$$TVC(t_1,T)=O_C+H_C+D_C+P_C$$
$$=\frac{A}{T}+\frac{1}{T}[C_1\{\frac{\alpha}{\theta}(e^{\theta(t_1-t_d)}-t_d)t_d-\frac{3\alpha t_d^{\ 2}}{2}-\frac{r\alpha t_d^{\ 2}}{2\theta}(e^{\theta(t_1-t_d)}-t_d)-\frac{r\alpha t_d^{\ 3}}{6}\}$$
$$+\{\frac{\alpha}{\theta}(-\frac{1}{\theta}+\frac{e^{\theta(t_1-t_d)}}{\theta}-(t_1-t_d))-\frac{r\alpha}{\theta}(-\frac{t_1}{\theta}+\frac{t_d e^{\theta(t_1-t_d)}}{\theta}-\frac{1}{\theta^2}+\frac{e^{\theta(t_1-t_d)}}{\theta^2}$$
$$-\frac{t_1^{\ 2}}{2}+\frac{t_d^{\ 2}}{2})\}]+\frac{C_d\theta}{T}[\{\frac{\alpha}{\theta}(-\frac{1}{\theta}+\frac{e^{\theta(t_1-t_d)}}{\theta}-(t_1-t_d))$$
$$-\frac{r\alpha}{\theta}(-\frac{t_1}{\theta}+\frac{t_d e^{\theta(t_1-t_d)}}{\theta}-\frac{1}{\theta^2}+\frac{e^{\theta(t_1-t_d)}}{\theta^2}-\frac{t_1^{\ 2}}{2}+\frac{t_d^{\ 2}}{2})\}]$$
$$+C_i[\frac{\alpha}{\theta}(e^{\theta(t_1-t_d)}-t_d)-(\alpha t_d)]$$

CASE II: When shortages are partially backlogged

The inventory level decreases only due to demand during the interval $[t_d, t_1]$. Hence the differential equation representing the inventory level is given by:

$$\frac{dI_1(t)}{dt}=-(\alpha+\beta t) \qquad 0\le t\le t_d \quad \ldots(13)$$

With the boundary condition $I_1(0)=I_{\max}$. Solution of equation (13) is:

$$I_1(t)=I_{\max}-(\alpha t+\frac{\beta t^2}{2}) \qquad 0\le t\le t_d \quad \ldots(14)$$

$$\frac{dI_2(t)}{dt}+\theta I_2(t)=-(\alpha+\beta t) \qquad t_d\le t\le t_1 \quad \ldots(15)$$

With the boundary condition $I_2(t_1)=0$. Solution of equation (15) is:

$$I_2(t)=[\frac{\alpha}{\theta}(e^{\theta(t_1-t)}-1)+\frac{\beta}{\theta}(t_1 e^{\theta(t_1-t)}-t)-\frac{\beta}{\theta^2}(e^{\theta(t_1-t)}-1)] \qquad t_d\le t\le t_1 \quad \ldots(16)$$

Equating the equation (14) and (16) at $t = t_d$

$$I_{\max} - (\alpha t_d + \frac{\beta t_d^{\,2}}{2}) = [\frac{\alpha}{\theta}(e^{\theta(t_1-t_d)} - t_d) + \frac{\beta}{\theta}(t_1 e^{\theta(t_1-t_d)} - t_d) - \frac{\beta}{\theta^2}(e^{\theta(t_1-t_d)} - 1)]$$

This implies that the maximum inventory level for each cycle is:

$$I_{\max} = [\frac{\alpha}{\theta}(e^{\theta(t_1-t_d)} - t_d) + \frac{\beta}{\theta}(t_1 e^{\theta(t_1-t_d)} - t_d) - \frac{\beta}{\theta^2}(e^{\theta(t_1-t_d)} - 1) - (\alpha t_d + \frac{\beta t_d^{\,2}}{2})] \quad \ldots(17)$$

Substitute the value of $I_{\max}$ from equation (17) in equation (14), we get

$$I_1(t) = [\frac{\alpha}{\theta}(e^{\theta(t_1-t_d)} - t_d) + \frac{\beta}{\theta}(t_1 e^{\theta(t_1-t_d)} - t_d) - \frac{\beta}{\theta^2}(e^{\theta(t_1-t_d)} - 1) - (\alpha t_d + \frac{\beta t_d^{\,2}}{2})] - (\alpha t + \frac{\beta t^2}{2})$$

$$0 \le t \le t_d \quad \ldots(18)$$

During the shortage interval $[t_1, T]$, the demand at any time t is partially backlogged at fraction . Thus the inventory level at any time t is governed by the following equation:

$$\frac{dI_3(t)}{dt} = -(\alpha + \beta t)e^{-\delta t} \qquad t_1 \le t \le T \quad \ldots(19)$$

$$I_3(t) = [\frac{\alpha}{\delta}(e^{-\delta t} - e^{-\delta t_1}) + \frac{\beta}{\delta}(te^{-\delta t} - t_1 e^{-\delta t_1}) - \frac{\beta}{\delta^2}(e^{-\delta t} - e^{-\delta t_1})] \quad t_1 \le t \le T \quad \ldots(20)$$

Substitute $t = T$ in equation (20), we obtain the maximum amount of demand backlogged per cycle as follows

$$S = (-I_3(t))$$

$$= [\frac{\alpha}{\delta}(e^{-\delta t_1} - e^{-\delta T}) - \frac{\beta}{\delta}(Te^{-\delta T} - t_1 e^{-\delta t_1}) + \frac{\beta}{\delta^2}(e^{-\delta T} - e^{-\delta t_1})] \quad \ldots(21)$$

From equation (17), we can obtain the order quantity Q as:

$$Q = I_{\max} + S$$

$$= [\frac{\alpha}{\theta}(e^{\theta(t_1-t_d)} - t_d) + \frac{\beta}{\theta}(t_1 e^{\theta(t_1-t_d)} - t_d) - \frac{\beta}{\theta^2}(e^{\theta(t_1-t_d)} - 1) - (\alpha t_d + \frac{\beta t_d^{\,2}}{2})$$

$$+ \frac{\alpha}{\delta}(e^{-\delta t_1} - e^{-\delta T}) - \frac{\beta}{\delta}(Te^{-\delta T} - t_1 e^{-\delta t_1}) + \frac{\beta}{\delta^2}(e^{-\delta T} - e^{-\delta t_1})] \quad \ldots(22)$$

The total relevant cost per unit time consists of

Ordering cost $O_C = \frac{A}{T}$ …(23)

Inventory holding cost $H_C = \frac{1}{T}[\int_0^{t_d} C_1 I(t) e^{-rt} dt + \int_{t_d}^{t_1} C_1 I(t) e^{-rt} dt]$

$$= \frac{1}{T}[C_1\{\frac{\alpha}{\theta}(e^{\theta(t_1-t_d)} - t_d)t_d + \frac{\beta}{\theta}(t_1 e^{\theta(t_1-t_d)} - t_d)t_d - \frac{\beta}{\theta^2}(e^{\theta(t_1-t_d)} - 1)t_d$$

$$-(\frac{3\alpha t_d^{\,2}}{2} + \frac{2\beta t_d^{\,3}}{3}) - \frac{r\alpha t_d^{\,2}}{2\theta}(e^{\theta(t_1-t_d)} - t_d) - \frac{r\beta t_d^{\,2}}{2\theta}(t_1 e^{\theta(t_1-t_d)} - t_d)$$

$$-\frac{r\beta t_d^{\,2}}{2\theta^2}(e^{\theta(t_1-t_d)} - 1) - \frac{rt_d^{\,2}}{2}(\alpha t_d + \frac{\beta t_d^{\,2}}{2}) + r(\frac{\alpha t_d^{\,3}}{3} + \frac{\beta t_d^{\,4}}{8})\}$$

$$+\{\frac{\alpha}{\theta}(-\frac{1}{\theta} + \frac{e^{\theta(t_1-t_d)}}{\theta} - (t_1 - t_d)) + \frac{\beta}{\theta}(-\frac{t_1}{\theta} + \frac{t_1 e^{\theta(t_1-t_d)}}{\theta} - \frac{t_1^{\,2}}{2} + \frac{t_d^{\,2}}{2})$$

$$-\frac{\beta}{\theta^2}(-\frac{1}{\theta} + \frac{e^{\theta(t_1-t_d)}}{\theta} - (t_1 - t_d)) - \frac{r\alpha}{\theta}(-\frac{t_1}{\theta} + \frac{t_d e^{\theta(t_1-t_d)}}{\theta} - \frac{1}{\theta^2} + \frac{e^{\theta(t_1-t_d)}}{\theta^2}$$

$$-\frac{t_1^{\,2}}{2} + \frac{t_d^{\,2}}{2}) - \frac{r\beta}{\theta}(-\frac{t_1^{\,2}}{\theta} + \frac{t_1 t_d e^{\theta(t_1-t_d)}}{\theta} - \frac{t_1}{\theta^2} + \frac{t_1 e^{\theta(t_1-t_d)}}{\theta^2} - \frac{t_1^{\,3}}{3} + \frac{t_d^{\,3}}{3})$$

$$+\frac{r\beta}{\theta^2}(-\frac{t_1}{\theta}+\frac{t_d e^{\theta(t_1-t_d)}}{\theta}-\frac{1}{\theta^2}+\frac{e^{\theta(t_1-t_d)}}{\theta^2}-\frac{t_1^{\ 2}}{2}+\frac{t_d^{\ 2}}{2})\}] \quad \ldots(24)$$

Inventory deterioration cost $D_C=\frac{C_d}{T}[\int_{t_d}^{t_1}\theta I(t)e^{-rt}dt]$

$$=\frac{C_d\theta}{T}[\{\frac{\alpha}{\theta}(-\frac{1}{\theta}+\frac{e^{\theta(t_1-t_d)}}{\theta}-(t_1-t_d))+\frac{\beta}{\theta}(-\frac{t_1}{\theta}+\frac{t_1e^{\theta(t_1-t_d)}}{\theta}-\frac{t_1^{\ 2}}{2}+\frac{t_d^{\ 2}}{2})$$

$$-\frac{\beta}{\theta^2}(-\frac{1}{\theta}+\frac{e^{\theta(t_1-t_d)}}{\theta}-(t_1-t_d))-\frac{r\alpha}{\theta}(-\frac{t_1}{\theta}+\frac{t_d e^{\theta(t_1-t_d)}}{\theta}-\frac{1}{\theta^2}+\frac{e^{\theta(t_1-t_d)}}{\theta^2}$$

$$-\frac{t_1^{\ 2}}{2}+\frac{t_d^{\ 2}}{2})-\frac{r\beta}{\theta}(-\frac{t_1^{\ 2}}{\theta}+\frac{t_1t_de^{\theta(t_1-t_d)}}{\theta}-\frac{t_1}{\theta^2}+\frac{t_1e^{\theta(t_1-t_d)}}{\theta^2}-\frac{t_1^{\ 3}}{3}+\frac{t_d^{\ 3}}{3})$$

$$+\frac{r\beta}{\theta^2}(-\frac{t_1}{\theta}+\frac{t_d e^{\theta(t_1-t_d)}}{\theta}-\frac{1}{\theta^2}+\frac{e^{\theta(t_1-t_d)}}{\theta^2}-\frac{t_1^{\ 2}}{2}+\frac{t_d^{\ 2}}{2})\}] \quad \ldots(25)$$

Purchase cost $P_C=C_iQ$

$$=C_i[\frac{\alpha}{\theta}(e^{\theta(t_1-t_d)}-t_d)+\frac{\beta}{\theta}(t_1e^{\theta(t_1-t_d)}-t_d)-\frac{\beta}{\theta^2}(e^{\theta(t_1-t_d)}-1)-(\alpha t_d+\frac{\beta t_d^{\ 2}}{2})$$

$$+\frac{\alpha}{\delta}(e^{-\delta t_1}-e^{-\delta T})-\frac{\beta}{\delta}(Te^{-\delta T}-t_1e^{-\delta t_1})+\frac{\beta}{\delta^2}(e^{-\delta T}-e^{-\delta t_1})] \quad \ldots(26)$$

Shortages cost $S_C=\frac{C_S}{T}[\int_{t_1}^{T}(-I(t))e^{-rt}dt]$

$$=\frac{C_S}{T}[(\frac{\alpha}{\delta}-\frac{\beta}{\delta^2})(e^{-\delta t_1}(T-t_1)+\frac{e^{-\delta T}}{\delta}-\frac{e^{-\delta t_1}}{\delta})+\frac{\beta}{\delta}(t_1e^{-\delta t_1}(T-t_1)+\frac{Te^{-\delta T}}{\delta}$$

$$-\frac{t_1e^{-\delta t_1}}{\delta}+\frac{e^{-\delta T}}{\delta^2}-\frac{e^{-\delta t_1}}{\delta^2})-\frac{\beta}{\delta^2}(e^{-\delta t_1}(T-t_1)+\frac{e^{-\delta T}}{\delta}-\frac{e^{-\delta t_1}}{\delta})$$

$$-\frac{\alpha r}{\delta}(\frac{(T^2-t_1^{\ 2})e^{-\delta t_1}}{2}+\frac{Te^{-\delta T}}{\delta}-\frac{t_1e^{-\delta t_1}}{\delta}+\frac{e^{-\delta T}}{\delta^2}-\frac{e^{-\delta t_1}}{\delta^2})$$

$$-\frac{\beta r}{\delta}(\frac{(T^2-t_1^{\ 2})t_1e^{-\delta t_1}}{2}+\frac{(T^2e^{-\delta T}-t_1^{\ 2}e^{-\delta t_1})}{\delta}+\frac{2}{\delta}(\frac{Te^{-\delta T}}{\delta}-\frac{t_1e^{-\delta t_1}}{\delta}+\frac{e^{-\delta T}}{\delta^2}-\frac{e^{-\delta t_1}}{\delta^2})$$

$$+\frac{r\beta}{\delta}(\frac{(T^2-t_1^{\ 2})e^{-\delta t_1}}{2}+\frac{Te^{-\delta T}}{\delta}-\frac{t_1e^{-\delta t_1}}{\delta}+\frac{e^{-\delta T}}{\delta^2}-\frac{e^{-\delta t_1}}{\delta^2})] \quad \ldots(27)$$

Lost sale cost $L_C=\frac{C_{LS}}{T}[\int_{t_1}^{T}(1-e^{-\delta t})(\alpha+\beta t)e^{-rt}dt]$

$$=\frac{C_{LS}}{T}[\alpha(-\frac{e^{-rT}}{r}+\frac{e^{-rt_1}}{r}+\frac{e^{-(r+\delta)T}}{(r+\delta)}-\frac{e^{-(r+\delta)t_1}}{(r+\delta)})+\beta(-\frac{Te^{-rT}}{r}+\frac{t_1e^{-rt_1}}{r}$$

$$-\frac{e^{-rT}}{r^2}+\frac{e^{-rt_1}}{r^2}+\frac{Te^{-(r+\delta)T}}{(r+\delta)}-\frac{t_1e^{-(r+\delta)t_1}}{(r+\delta)}-\frac{e^{-(r+\delta)T}}{(r+\delta)^2}+\frac{e^{-(r+\delta)t_1}}{(r+\delta)^2}] \quad \ldots(28)$$

The total relevant inventory cost per unit time can be formulated as:

$$TVC(t_1,T)=O_C+H_C+D_C+P_C+S_C+L_C$$

$$=\frac{A}{T}+\frac{1}{T}[C_1\{\frac{\alpha}{\theta}(e^{\theta(t_1-t_d)}-t_d)t_d+\frac{\beta}{\theta}(t_1e^{\theta(t_1-t_d)}-t_d)t_d-\frac{\beta}{\theta^2}(e^{\theta(t_1-t_d)}-1)t_d$$

$$-(\frac{3\alpha t_d^{\ 2}}{2}+\frac{2\beta t_d^{\ 3}}{3})-\frac{r\alpha t_d^{\ 2}}{2\theta}(e^{\theta(t_1-t_d)}-t_d)-\frac{r\beta t_d^{\ 2}}{2\theta}(t_1e^{\theta(t_1-t_d)}-t_d)$$

$$-\frac{r\beta t_d^{\,2}}{2\theta^2}(e^{\theta(t_1-t_d)}-1)-\frac{rt_d^{\,2}}{2}(\alpha t_d+\frac{\beta t_d^{\,2}}{2})+r(\frac{\alpha t_d^{\,3}}{3}+\frac{\beta t_d^{\,4}}{8})\}$$
$$+\{\frac{\alpha}{\theta}(-\frac{1}{\theta}+\frac{e^{\theta(t_1-t_d)}}{\theta}-(t_1-t_d))+\frac{\beta}{\theta}(-\frac{t_1}{\theta}+\frac{t_1e^{\theta(t_1-t_d)}}{\theta}-\frac{t_1^{\,2}}{2}+\frac{t_d^{\,2}}{2})$$
$$-\frac{\beta}{\theta^2}(-\frac{1}{\theta}+\frac{e^{\theta(t_1-t_d)}}{\theta}-(t_1-t_d))-\frac{r\alpha}{\theta}(-\frac{t_1}{\theta}+\frac{t_de^{\theta(t_1-t_d)}}{\theta}-\frac{1}{\theta^2}+\frac{e^{\theta(t_1-t_d)}}{\theta^2}$$
$$-\frac{t_1^{\,2}}{2}+\frac{t_d^{\,2}}{2})-\frac{r\beta}{\theta}(-\frac{t_1^{\,2}}{\theta}+\frac{t_1t_de^{\theta(t_1-t_d)}}{\theta}-\frac{t_1}{\theta^2}+\frac{t_1e^{\theta(t_1-t_d)}}{\theta^2}-\frac{t_1^{\,3}}{3}+\frac{t_d^{\,3}}{3})$$
$$+\frac{r\beta}{\theta^2}(-\frac{t_1}{\theta}+\frac{t_de^{\theta(t_1-t_d)}}{\theta}-\frac{1}{\theta^2}+\frac{e^{\theta(t_1-t_d)}}{\theta^2}-\frac{t_1^{\,2}}{2}+\frac{t_d^{\,2}}{2})\}]$$
$$+\frac{C_d\theta}{T}[\{\frac{\alpha}{\theta}(-\frac{1}{\theta}+\frac{e^{\theta(t_1-t_d)}}{\theta}-(t_1-t_d))+\frac{\beta}{\theta}(-\frac{t_1}{\theta}+\frac{t_1e^{\theta(t_1-t_d)}}{\theta}-\frac{t_1^{\,2}}{2}+\frac{t_d^{\,2}}{2})$$
$$-\frac{\beta}{\theta^2}(-\frac{1}{\theta}+\frac{e^{\theta(t_1-t_d)}}{\theta}-(t_1-t_d))-\frac{r\alpha}{\theta}(-\frac{t_1}{\theta}+\frac{t_de^{\theta(t_1-t_d)}}{\theta}-\frac{1}{\theta^2}+\frac{e^{\theta(t_1-t_d)}}{\theta^2}$$
$$-\frac{t_1^{\,2}}{2}+\frac{t_d^{\,2}}{2})-\frac{r\beta}{\theta}(-\frac{t_1^{\,2}}{\theta}+\frac{t_1t_de^{\theta(t_1-t_d)}}{\theta}-\frac{t_1}{\theta^2}+\frac{t_1e^{\theta(t_1-t_d)}}{\theta^2}-\frac{t_1^{\,3}}{3}+\frac{t_d^{\,3}}{3})$$
$$+\frac{r\beta}{\theta^2}(-\frac{t_1}{\theta}+\frac{t_de^{\theta(t_1-t_d)}}{\theta}-\frac{1}{\theta^2}+\frac{e^{\theta(t_1-t_d)}}{\theta^2}-\frac{t_1^{\,2}}{2}+\frac{t_d^{\,2}}{2})\}]$$
$$+C_i[\frac{\alpha}{\theta}(e^{\theta(t_1-t_d)}-t_d)+\frac{\beta}{\theta}(t_1e^{\theta(t_1-t_d)}-t_d)-\frac{\beta}{\theta^2}(e^{\theta(t_1-t_d)}-1)-(\alpha t_d+\frac{\beta t_d^{\,2}}{2})$$
$$+\frac{\alpha}{\delta}(e^{-\delta t_1}-e^{-\delta T})-\frac{\beta}{\delta}(Te^{-\delta T}-t_1e^{-\delta t_1})+\frac{\beta}{\delta^2}(e^{-\delta T}-e^{-\delta t_1})]$$
$$+\frac{C_S}{T}[(\frac{\alpha}{\delta}-\frac{\beta}{\delta^2})(e^{-\delta t_1}(T-t_1)+\frac{e^{-\delta T}}{\delta}-\frac{e^{-\delta t_1}}{\delta})+\frac{\beta}{\delta}(t_1e^{-\delta t_1}(T-t_1)+\frac{Te^{-\delta T}}{\delta}$$
$$-\frac{t_1e^{-\delta t_1}}{\delta}+\frac{e^{-\delta T}}{\delta^2}-\frac{e^{-\delta t_1}}{\delta^2})-\frac{\beta}{\delta^2}(e^{-\delta t_1}(T-t_1)+\frac{e^{-\delta T}}{\delta}-\frac{e^{-\delta t_1}}{\delta})$$
$$-\frac{\alpha r}{\delta}(\frac{(T^2-t_1^{\,2})e^{-\delta t_1}}{2}+\frac{Te^{-\delta T}}{\delta}-\frac{t_1e^{-\delta t_1}}{\delta}+\frac{e^{-\delta T}}{\delta^2}-\frac{e^{-\delta t_1}}{\delta^2})$$
$$-\frac{\beta r}{\delta}(\frac{(T^2-t_1^{\,2})t_1e^{-\delta t_1}}{2}+\frac{(T^2e^{-\delta T}-t_1^{\,2}e^{-\delta t_1})}{\delta}+\frac{2}{\delta}(\frac{Te^{-\delta T}}{\delta}-\frac{t_1e^{-\delta t_1}}{\delta}+\frac{e^{-\delta T}}{\delta^2}-\frac{e^{-\delta t_1}}{\delta^2})$$
$$+\frac{r\beta}{\delta}(\frac{(T^2-t_1^{\,2})e^{-\delta t_1}}{2}+\frac{Te^{-\delta T}}{\delta}-\frac{t_1e^{-\delta t_1}}{\delta}+\frac{e^{-\delta T}}{\delta^2}-\frac{e^{-\delta t_1}}{\delta^2})]$$
$$+\frac{C_{LS}}{T}[\alpha(-\frac{e^{-rT}}{r}+\frac{e^{-rt_1}}{r}+\frac{e^{-(r+\delta)T}}{(r+\delta)}-\frac{e^{-(r+\delta)t_1}}{(r+\delta)})+\beta(-\frac{Te^{-rT}}{r}+\frac{t_1e^{-rt_1}}{r}$$
$$-\frac{e^{-rT}}{r^2}+\frac{e^{-rt_1}}{r^2}+\frac{Te^{-(r+\delta)T}}{(r+\delta)}-\frac{t_1e^{-(r+\delta)t_1}}{(r+\delta)}-\frac{e^{-(r+\delta)T}}{(r+\delta)^2}+\frac{e^{-(r+\delta)t_1}}{(r+\delta)^2}] \quad \ldots(29)$$

To minimize the total average cost per unit of time, the optimal value of t_1 and T can be obtained by the following equations

$$\frac{\partial TVC(t_1,T)}{\partial t_1}=0 \qquad \text{and} \qquad \frac{\partial TVC(t_1,T)}{\partial T}=0$$

$$\frac{\partial^2 TVC(t_1,T)}{\partial^2 t_1} > 0 \quad \text{and} \quad \frac{\partial^2 TVC(t_1,T)}{\partial^2 T} > 0$$

IV. SPECIAL CASES

CASE A: When $\beta = 0$, demand rate is constant.

V. CONCLUSION

An inventory model of determining the optimal replenishment policy for non-instantaneous deteriorating items with linear demand has been developed. In many cases customers are conditioned to a shipping delay, and may be willing to wait for a short time in order to get their first choice. Generally speaking, the length of the waiting time for the next replenishment is the main factor for deciding whether the backlogging will be accepted or not. The willingness of a customer to wait for backlogging during a shortage period declines with the length of the waiting time. Thus, inventory shortages are allowed and partially backordered in the present chapter and the backlogging rate is considered as a decreasing function of the waiting time for the next replenishment. The main goal of this study is to minimize the total average cost. The model developed may further be extended for production inventory model, inflation and permissible delay in payments.

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